CLAYTON STATE UNIVERSITY

Center for Academic Success

Library-Lower Level • (678) 466-4070
http://adminservices.clayton.edu/cas

Mathematics Success Tips Series: Factoring – Basic principles

Always try to factor out a greatest common factor (GCF) first.

Doing this will make the rest of the problem easier and save you time. This style of factoring is the reverse of distributing.

Example 1: Factor $8a+2ab+10b^3$. Answer 1: $2(4a+ab+5b^3)$.

Solution 1: To do this find the largest number and variable that will divide evenly into all the terms called the GCF. For this example, 2 is the largest number that is a factor and there are no common variables so the GCF is 2. Write the GCF and then, in parentheses, write what is left from each term after dividing by the GCF. So $8a+2ab+10b^3$ becomes $2(4a+ab+5b^3)$.

Example 2: Factor $3x^2y + 12xy^6 – 6x^2 y^2+3xy$. Answer 2: $3xy(x+4y^5– 2xy+1)$.

Practice 3: Factor $5a^3b^2 – 20a^2b^6 + 15ab^8$. You try to solve this one.

The next step will be different depending on the number of terms of the polynomial. If there are:

- ## 4 terms: Try to factor by grouping.

This method starts by grouping sets of two terms together. It is usually fine to group the first two terms and the last two terms, although there are some problems where it is easier to do a different grouping where terms that have similar variables are grouped together.

Example 4: Factor 50wy – 100wz + 3xy – 6xz . Answer 4: (y –2z)(50w+3x).

Solution 4: Start by working with the first two terms and factor out their GCF. So for our problem, 50wy – 100wz becomes 50w(y –2z). Then do the same for the last to terms keeping the sign from the third term so 3xy – 6xz becomes +3x(y –2z). So we have 50wy – 100wz + 3xy – 6xz = 50w(y –2z) + 3x(y –2z). You now have two terms (they're large but there's still only two). The expression in the parenthesis should be exactly the same and is the GCF of these two large terms. If the expression in the parenthesis is not the same then either you've made a mistake (you should try again) or the problem does not factor with this method. Now factor out this new GCF. So 50wy – 100wz + 3xy – 6xz = 50w(y –2z) + 3x(y –2z) = (y –2z)(50w+3x).

Important Note: When the third term is negative students often get confused on the signs, so let's do Example 4 again with a few signs changed. Factor: 50wy – 100wz – 3xy + 6xz. We use the same process but when we factor the GCF out of the last two terms we use – 3x which will change the sign on the last term. So 50wy – 100wz – 3xy + 6xz = 50w(y – 2z) – 3x(y – 2z)=(y – 2z)(50w – 3x).

Example 5: Factor rw – tw + 2r – 2t. Answer 5: (r –t)(w+2).

Practice 6: Factor $s^2q^2 + q^2 +rs^2 + r$. You try to solve this one.

- **3 terms: Try the product/sum method or trial and error.**

The product/sum method and trial and error both result in the same answer. The product/sum method is more systematic and easier to follow but will take a little longer. The trial and error is intuitive and more difficult but faster. You may use either method, whichever works for you. You may prefer the product/sum method in the beginning, but as you get better at factoring the trial and error method might be faster. These styles of factoring are the reverse of “FOIL”.

Example 7a: Factor $3x^2+14x-5$ with the product/sum method. Answer 7a: (3x-1)(x+5).

Solution 7a: Take the leading coefficient (the “a” from the ax^2+bx+c form) and multiply it by the last term (the “c” from the ax^2+bx+c form). This new number is the product we are looking for. For our problem we use (3)(-5)=-15 as the product. The sum we are looking for is the middle coefficient (the “b” from the ax^2+bx+c form). So for our problem the sum is 14. The list all the possible combinations of 2 numbers that multiply to be the product. So -15 = (1)(-15), (-1)(15), (-3)(5), (3)(-5). Then add each of these 2 numbers to see which adds to our sum number 1 + -15 = -14, -1+15 = 14, -3+5 = 2, 3 +-5 = -2. So the numbers we are looking for are -1 and 15 because they multiply to our product of -15 and they add to our sum of 14. If you can see which two numbers work right away you don’t have to try all the combinations you can just skip ahead, but remember to be careful about the signs.

Once you have the two numbers you rewrite the problem using the two new numbers for the linear term. So $3x^2+14x-5$ becomes $3x^2-1x+15x-5$. Now the problem has 4 terms so we factor by grouping as we did in the previous section. $3x^2+14x-5 = 3x^2-1x+15x-5 = x(3x-1)+5(3x-1) = (3x-1)(x+5)$.

Example 7b: Factor $3x^2+14x-5$ with the trial and error method. Answer 7a: (3x-1)(x+5).

Solution 7b: Try all the possible combinations where the first term in each parentheses multiply to be the leading term and the last term in each parentheses multiply to be the last term of the problem. So for our problem the possibilities are (3x-5)(x+1), (3x+5)(x-1), (3x+1)(x-5), (3x-1)(x+5). We multiply (FOIL) to see which one is the correct result: $(3x-5)(x+1) = 3x^2-2x-5$, $(3x+5)(x-1) = 3x^2+2x-5$, $(3x+1)(x-5) = 3x^2-14x-5$, $(3x-1)(x+5) = 3x^2+14x-5$. So we see that the only correct result is from (3x-1)(x+5). As you get better at seeing the patterns and guessing you won’t have to write all the possibilities out and will hopefully try the right combination the first or second try each time.

Example 8: Factor $6x^2-10xy-4y^2$. Answer 8: (2x-4y)(3x+y).

Practice 9: Factor $6x^2 – 11x – 10$. You try to solve this one.

- **2 terms: Try a special formula.**

There are special formulas for the differences of perfect squares and for the sums and differences of perfect cubes. There are also special formulas for perfect square trinomials.

$a^2 - b^2 = (a-b)(a+b)$ $\quad$ $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ $\quad$ $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$* \, a^2 + 2ab + b^2 = (a+b)(a+b)$ $\quad$ $* \, a^2 - 2ab + b^2 = (a-b)(a-b)$

*Notice that these two formulas have three terms, that means that instead of using the formula you can get the answer using the product/sum method or the trial and error method from the previous section, however if you can recognize the special formulas it will be faster.

Example 10: Factor $8x^3 - 27$. Answer 10: $(2x-3)(4x^2+6x+9)$.

Solution 10: Check to see if the exponents are squares or cubes. Then rewrite each term as a perfect square or cube depending on which case you are in. So in our example we have cubes and $8x^3 = (2x)^3$ and $27 = (3)^3$. So we rewrite $8x^3-27$ as $(2x)^3 - (3)^3$. Now "a" is the expression in the first parenthesis (2x for us) and "b" is the expression in the second parenthesis, (3 for us). Apply the formula to see

$8x^3 - 27 = (2x)^3 - (3)^3 = (2x-3)((2x)^2 + (2x)(3) + (3)^2) = (2x-3)(4x^2+6x+9)$.

Example 11: Factor $9x^2 - 64$. Answer 11: (3x-8)(3x+8).

Practice 12: Factor $y^3 + 64$. You try to solve this one.

After each successful factorization try to factor again according to the principles listed above until no further factorization is possible. If the polynomial was not able to be factored at all (not even with a GCF) then it is called *prime.*

Putting it all together. Many problems involve several different steps of factoring. Just take them one step at a time and follow the basic principles.

Example 13: Factor $24x^3y + 12x^2y - 54xy - 27y$. Answer 13: $3y(2x+1)(2x-3)(2x+3)$.

Solution 13: First, remember to always try to factor out a GCF. Here the GCF is 3y so $24x^3y + 12x^2y - 54xy - 27y =$ $3y(8x^3 + 4x^2 - 18x - 9)$. Now we will leave the GCF alone in front of the problem for the rest of the time. Inside the parentheses there are 4 terms so we factor by grouping. This gives $8x^3 + 4x^2 - 18x - 9 = (2x+1)(4x^2-9)$. So we now have $3y(2x+1)(4x^2-9)$. The (2x+1) is linear now so we're done with it, but the $(4x^2-9)$ can still be factored. It has 2 terms so we use a special formula, the difference of squares. $4x^2-9 = (2x-3)(2x+3)$. So we now arrive at the final answer, $3y(2x+1)(2x-3)(2x+3)$.

Now here are several problems for you to practice. The instructions for all of them are to factor.

1. $9b^4 - 11b^3 + 12b^2$
2. $n + nx + m + mx$
3. $50wy - 100wz + 50xy - 100xz$
4. $ad^2 - 25a + bd^2 - 25b$
5. $16m^4n^2 + 20m^2n^4 - 32mn^8$
6. $p^3 - 5p^2 - 2p + 10$
7. $awy + 6awz + axy + 6axz$
8. $x^2 - 26x + 48$
9. $a^2 + 3a - 28$
10. $x^2 - 13x + 42$
11. $x^2 - 18x - 40$
12. $x^2 - x - 30$
13. $2x^2 - x - 3$
14. $6x^2 - 11x - 10$
15. $6x^2 + x - 2$
16. $a^2 - 6a - 40$
17. $4a^2 + 5a - 6$
18. $9x^2 - 3x - 2$
19. $5a^2 + 5a - 30$
20. $x^4 + 5x^3 - 24x^2$
21. $6x^2 + 17x + 12$
22. $m^3 + 6m^2 - 20m$
23. $50y^5 + 400y^4 + 350y^3$
24. $20y^2 + 7y - 6$
25. $18y^2 + 31y + 6$
26. $15x^2 + 4x - 4$
27. $10w^2 - 9w - 7$
28. $x^2 + 10x + 25$
29. $x^2 - 49$
30. $n^2 - 20n + 100$
31. $x^2 - 25$
32. $m^2 + 8m + 16$
33. $x^2 - 8xy + 16y^2$
34. $m^2 - 49y^2$
35. $x^2 - 81y^2$
36. $y^2 - 12y + 36$
37. $x^2 + 16x + 64$
38. $m^2 - 6mn + 9n^2$
39. $m^3 + 27$
40. $4x^2 + 20x + 25$
41. $x^3 + 8y^3$
42. $100m^2 - 4n^2$
43. $x^3 - 8$
44. $8x^3 - 27y^3$
45. $27m^3 + 8n^3$
46. $8x^3 - 1000$
47. $27m^3 + 64n^3$
48. $14x^2 + 41x + 15$
49. $36x^2 - 3x - 5$
50. $x^3 - 11x^2 - 126x$
51. $35x^2 - 34x + 8$
52. $42x^2 + 29x - 5$
53. $8x^3 + 46x^2 + 60x$
54. $x^2 + 8x - 240$
55. $3x^2 - 34x + 75$
56. $15x^2 - 14x - 16$
57. $16x^2y + 50xy - 21y$
58. $45x^2 + 59x + 6$
59. $x^2 - 21x + 80$
60. $36x^2 + x - 21$
61. $24x^2 - 46x + 21$
62. $3x^2 + 48x + 144$
63. $33x^2 - 82x - 16$